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Odds ratio: an ecologically sound tool to compare proportions

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The correct use and interpretation of statistical measures is often challenging for field-oriented ecologists. One such basic measure is the odds ratio (OR), which enables the comparison of two proportions. Odds ratio is the pivotal concept in the simple analyses of proportions in two-way contingency tables, as well as in complex logit model approaches. Here, we clarify the use and interpretation of the odds ratio in ecological research. We show that the odds ratio is both a statistical and an ecological solution to quantifying the direction and magnitude of discrepancy between proportions. To enhance comparison of suppressing (with OR below one) and promoting (with OR above one) factors, we propose that the odds ratio should always be reported as a value above one, together with an exponent (1 or –1) to denote the direction of the effect. The odds ratio supports powerful ecological interpretations in the comparison of proportions and thus should become a standard concept in ecological papers.

Introduction

Many natural phenomena can only be described using binary classifications, such as the presence or absence of a threatened species on decaying trunks, or an individual being a male or a female. The behaviour of dichotomies is described using proportions. Classical statistical techniques like analysis of (co)variance and linear regression are not appropriate for comparing proportions, because the underlying dichotomy leads to a binomial error distribution and, consequently, to violation of normally distributed errors with homogenous variances inherent in these methods (Sokal & Rohlf 1995). The general solution to these technical problems is provided by binary regression models (Collett 2002). In this paper, we will concentrate on one specific group of models with dichotomous response, i.e. logit models, which are widely used in ecology (for other alternatives to model dichotomous responses *see* Collett 2002).

A central tool to compare proportions is *Odds Ratio* (OR), which takes into account their special feature as quantities whose values are restricted to the interval [0, 1], or [0, 100] if percentages are used (Agresti 2002). OR is a standard part of the output in statistical pack-

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ages, and the increasing use of logit models, particularly logistic regression, has introduced OR to the ecological literature (e.g. Carignan & Villard 2002, Frappier & Eckert 2003, Kittredge *et al*. 2003). In the medical literature, the use and misuse of OR has received considerable attention (Walter 2000, Holcomb *et al*. 2001), but we feel that not too many ecologists are fully aware of what OR actually represents. Our aim is to clarify the interpretation of OR by referring to ecological substance and skipping technical aspects of statistical testing.

Odds ratio quantifies comparison of proportions

An imaginary fungus *Imaginarius adhoccius* occurs on fallen trunks of Norway spruce and silver birch. An ecological question of interest is: does the species prefer spruce trunks to birch trunks? In a nature reserve in eastern Finland, the species occurred on 10% and 5% of spruce and birch trunks, respectively (Table 1). Based on these proportions, many ecologists might conclude that *I. adhoccius* occurs more frequently on spruce trunks. According to ecological practice, this observation has to be accompanied with a significant result in the χ^2-test ($\chi^2 = 5.794$, d.f. $= 1$, $P = 0.016$ in our example), otherwise claims about the discrepancy between the proportions are not taken seriously. This additional statistical requirement does not, however, say anything about the *amount* of discrepancy between the two proportions — the interpretation remains purely qualitative, as statistical significance is either achieved or not. Consequently, statistical and ecological significance seem to be taken as more or less the same thing (*see* Yoccoz 1991 for a similar discussion on quantitative variables). In analogy to the evaluation of the mean values in ANOVA or linear regression, we need an appropriate measure of discrepancy between two proportions that enables quantitative *ecological* interpretations. Odds ratio is such a measure as it carries information on both the direction and size of the effect, and in certain situations, as will be seen, it is superior to other measures (e.g. difference or ratio of the proportions under comparison).

The conclusions concerning two proportions may focus on one or both of the following two aspects: the absolute values of the proportions and their comparison. A forest pathologist, who is studying the ability of trees to resist *I. adhoccius* infection, might be interested in the values of the proportions themselves. But a conservation ecologist, deciding whether to restore birch or spruce trunks to better promote the species, might rather focus on the discrepancy between the two proportions. When the odds ratio is used, the focus is on comparison. This is emphasized by the fact that from a given value of OR, one cannot reconstruct the original proportions under comparison; hence, they have to be reported separately. This is in analogy to comparing two quantities using their difference: from the difference itself (e.g. 5) one cannot tell the components: for example, $10 - 5 = 50 - 45$.

Odds ratio is the ratio of two odds

First, to calculate an odds ratio, the proportions under comparison are transformed into *odds*. The odds corresponding to a proportion p is defined as $p/(1 - p)$, i.e. the ratio of the probability that an incident takes place to the probability that it does not. The odds that a spruce trunk is occupied by the fungus is thus calculated as 0.10/(1

Table 1. The frequency of spruce and birch trunks with and without the imaginary fungus *Imaginarius adhoccius*.

Presence of the species	Spruce	Birch	Total
Yes	$n_{11} = 14$	$n_{21} = 50$	$n_{+1} = 64$
No	$n_{12} = 126$	$n_{22} = 950$	$n_{+2} = 1076$
Total	$n_{1+} = 140$	$n_{2+} = 1000$	$n_{Total} = 1140$
Proportion	$p_S = n_{11}/n_{1+} = 14/140 = 10.00\%$	$p_B = n_{21}/n_{2+} = 50/1000 = 5.00\%$	$p_{Total} = n_{+1}/n_{Total} = 64/1140 = 5.61\%$

– 0.10) = 0.111 (Table 1); for birch trunks the odds are 0.052 [= 0.05/(1 – 0.05)]. Second, to compare the two proportions (p, r), their odds are compared by using their ratio, which is consequently called the *odds ratio*: OR(p, r) = odds (p)/odds(r); here, the focal proportion p is compared to the reference proportion r. The odds for spruce trunks to be occupied by the fungus are 2.111 times as large as the odds for birch trunks: 0.111/0.052 = 2.111. In other words, the odds increase by 111% when birch trunks are replaced with spruce trunks. If OR = 1, the proportions under comparison are equal. Note that the odds ratio is *not* the ratio of two probabilities, which is known as the *risk ratio*, but the ratio of two odds.

Often when we observe two proportions, we — unconsciously — compare them as 'normal' numbers, i.e. by their difference. This approach takes us into problems as it does not take into account the fact that proportions range within the interval [0, 1]. For example, 10% is 5 percentage points greater than 5%, but which proportion exceeds 96% by as much as 10% exceeds 5%? The obvious answer (96 + 5) is above 100% and cannot serve as a proportion. The same problem arises when proportions are compared using their ratio. Multiplying 60% by 2 (= 0.10/0.05) results in 120%, which is not a proportion. But when the odds ratio is used, the calculations always result in a genuine proportion. Starting from r = 96%, the proportion p has odds 2.111 × 0.96/(1 – 0.96) = 50.67; thus, p = 50.67/(1 + 50.67) = 98.1%; starting from r = 60%, p has odds 2.111 × 0.60/(1 – 0.60) = 3.167; thus p = 3.167/(1 + 3.167) = 76.0%. The discrepancies between the proportions in the three pairs 5% and 10%, 60% and 76%, as well as 96% and 98% are all equal when measured as odds ratios.

The odds ratio can be calculated using frequencies only, without explicit reference to the corresponding proportions. The formula is OR = $(n_{11}/n_{12})/(n_{21}/n_{22})$, or in a more compact form as a *cross-product ratio* $(n_{11} \times n_{22})/(n_{12} \times n_{21})$ = (14 × 950)/(126 × 50) = 2.111 (Table 1). Although these formulas are simple in the sense that they directly use the frequencies, they sometimes seem to hide the essence that odds ratio is a tool to compare *proportions*, which in this example are 10% = 14/140 and 5% = 50/1000. In addition, the roles of the focal (p) and reference (r) proportion are not necessarily stated clearly (*see* below). From a statistical point of view, the frequencies are important as they determine the standard error of the estimated proportion: an estimated proportion, e.g. 5%, is more precise if it is based on frequencies 50 and 1000 than on 5 and 100, or even 1 and 20. But the ecological interpretation is ultimately based on proportions and their discrepancy. When odds ratios are estimated from models with several explanatory factors, explicit simple formulas — like in the one factor case above — do not necessarily exist; instead, iterative algorithms are needed.

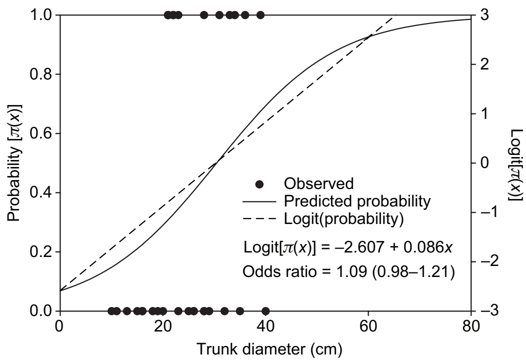

Fig. 1. The probability of fungal occupancy as a function of trunk diameter (cm). On the probability scale (left vertical axis) the curve is sigmoidal whereas on the logistic scale (right vertical axis) it is linear. Observed data are coded as 1 (occupied) and 0 (not occupied). For illustrative reasons, only values between –3 and 3 of logit[$\pi(x)$] are shown, although it may have any number as its value.

Odds ratio and quantitative explanatory variables

Odds ratios can also be used to characterize the effect of quantitative explanatory variables on binary responses. Hence, OR's are a standard part of the output of such *logistic regression* models. In our second example (Fig. 1), trunk diameter is used to explain variation in the proportion of *I. adhoccius* occupied trunks. The range of values from 10 to 40 cm results in 31 potential values for which the proportion of trunks with fruiting bodies is observed. Among these proportions, there are 465 (= 31 × 30/2) different pairs for which the two proportions can be compared. The results of these comparisons can, however, be reduced to a single odds ratio as follows.

First, take a fixed value of the quantitative explanatory variable, e.g. trunk diameter 20 cm, and denote by $\pi(20)$ the probability that a randomly chosen trunk of diameter 20 cm is occupied by *I. adhoccius*. Next, take another trunk diameter, e.g. 21 cm, and compare $\pi(21)$ to $\pi(20)$ using OR. The resulting value characterizes the effect of a 1 cm increase in trunk diameter on the proportion of trunks occupied by *I. adhoccius*. If we have reason to believe that this effect is independent of the starting value 20 cm, we may characterize all pairs of proportions with 'distance' 1 cm using a single value of OR. Heuristically, such a value is obtained as the average across values of x of OR's from comparisons of $\pi(x + 1)$ with $\pi(x)$, i.e. $\pi(11)$ with $\pi(10)$; $\pi(12)$ with $\pi(11)$ and so on up to comparing $\pi(40)$ with $\pi(39)$. This is exactly what the estimated effect of trunk diameter as odds ratio gives in a logistic regression output.

In the example, OR characterizing the effect of a 1 cm increase in trunk diameter is estimated to be 1.089 with 95% confidence interval [0.98, 1.21]. From this it is easy to calculate the effect of increase of any size in trunk diameter. As for each increase of 1 cm, the odds increase 1.089-fold, the increase for e.g. 2 centimetres is 1.089^2 = 1.186-fold and for 10 centimetres 1.089^{10} = 2.346-fold.

The assumption that the effect of a 1 cm increase, measured as OR = OR(x + 1, x) = odds[$\pi(x + 1)$]/odds[$\pi(x)$], is independent of the starting value x implies that

$$\ln(\text{OR}) = \ln\{\text{odds}[\pi(x + 1)] / \text{odds}[\pi(x)]\}$$
$$= \text{logit}[\pi(x + 1)] - \text{logit}[\pi(x)] \quad (1)$$

where logit[$\pi(x)$] = ln odds[$\pi(x)$]. This further implies that logit[$\pi(x)$] depends linearly on the value of diameter (x) (Fig. 1):

$$\text{logit}[\pi(x)] = \alpha + \beta x \quad (2)$$

where β = logit[$\pi(x + 1)$] – logit[$\pi(x)$]. Hence OR = exp(β). [Due to this, some statistical programs report OR under the heading exp(B)]. Positive values of β result in odds ratios above 1, indicating an increase of proportion; negative values, below 1. In the example (Fig. 1), OR = 1.089 = exp(0.0856). For a 1 cm *decrease* in trunk diameter, the odds change $\exp(-\beta) = [\exp(\beta)]^{-1}$-fold. Change of the sign results in an inverse value of the corresponding odds ratio: exp(–0.08557) = 0.918; the latter equating 1.089^{-1}. Absolute values of occupation probabilities for each diameter $\pi(x)$ are obtained as:

$$\pi(x) = \exp(\alpha + \beta x)/[1 + \exp(\alpha + \beta x)] \quad (3)$$

Focal proportion should be compared with reference proportion

When two proportions are compared, it makes a difference in which order the comparison is performed, i.e. which one of the proportions is used as a reference. Because OR is calculated as a ratio, a change of the comparison order of the proportions results in an inverse value of OR. A comparison of p = 10% with r = 5% results in OR = 2.111, whereas a comparison of 5% with 10% leads to OR = 0.473 (which is the inverse of 2.111). Both of these odds ratios are calculated from the same pair of proportions, and thereby they reflect the same amount of discrepancy, but in opposite directions. Table 2 clarifies this fact: a value of OR and its inverse are located symmetrically with respect to the main diagonal. Unfortunately, this fact is not too clear for most of us.

Our experience is that odds ratios below 1 are more difficult to interpret than those above it. This is perhaps due to the fact that from below, odds ratios are limited by 0, whereas from above, there is no limit. The neutral value (i.e. 1) of an odds ratio can thus decrease only by 100%, whereas its increase is not limited by any percentage. Consequently, the comparison of *two* odds ratios, one below and the other above 1, is difficult, as they are represented on a different scale. It would thus be desirable to have only odds ratios whose values are on the same scale, e.g. above 1. There are several ways to force the value of the odds ratio to be above 1, the simplest being perhaps the change of the roles of focal and reference proportions. These roles are, however, often determined by substance matter arguments, i.e. there exists a natural reference class. It is, for example, customary to compare the new, present or experimental conditions to the old, previous or reference ones. Sometimes changes in wording, e.g. from "shade tolerance" to "shade intolerance" can do the trick, but may contradict customary use of terms. For a quantitative explanatory variable *x*, the odds corresponding to a larger value of *x* (say, $x + 1$) are compared with the odds at a smaller value *x*. Here, changing the order of comparison seems very unnatural. Change of the roles of occurrence and non-occurrence in the response (i.e. reporting proportions of absence instead of presence) leads to inverse value of the odds ratio as well. These methods cannot, however, be used in multivariate phenomena, where both promoting (OR > 1) and suppressing (OR < 1) factors for a common response variable are present. Next we will propose a technique that will enable comparison of promoting and suppressing factors with respect to their effect sizes.

Exponent –1 or 1 denotes the direction of comparison

If the reported value of OR in computer output is below one, we suggest that it is replaced with its inverse value (that will now exceed 1) in the research report. To report the desired or natural direction of the comparison, the exponent –1 is used. For example, if it is natural to compare 5% with 10% (i.e. a decrease has taken place) we will not report the value of the corresponding OR = 0.474; instead, we write 2.111^{-1}. In this way important information about the direction of change is retained; yet the values of odds ratios are readily comparable, even when the directions of chance are opposite. Reporting changes as percentages do not have this property as the value 0.474 corresponds to a decrease of 52.6%, whereas its inverse 2.111 corresponds to an increase of 111%.

The inverse representation enables us to use natural references and customary terming without loosing comparability. It is especially useful in multiple logistic regression and designs with several factors as it enables comparison of effect sizes of suppressing and promoting factors (Table 3; modified from Frappier & Eckert 2003): The effect of the factor 'Conifer' may seem negligible because of the small value 0.001. Use of the inverse presentation reveals that it is the most important determinant of all (OR = 1000^{-1} = 0.001). (Actually, all values between 2000^{-1} and 671^{-1} are compatible with the rounded value 0.001.) The effect of 'Fertility requirement' is 0.018. Only after representing it as 55.6^{-1} we may see that it somewhat exceeds the effect of

Table 2. Odds ratio (odds of *p* divided by odds of *r*) as a function of the focal (*p*) and reference (*r*) proportions. Values of the odds ratio that are located symmetrically with respect to the main diagonal are inverses of each other, e.g. 2.11^{-1} = 0.47 or 4.75^{-1} = 0.21 (*see* text for details). Note also that the odds ratio is symmetrical with respect to focal and reference proportions; for example, the last row (*p* = 0.95) and first column (*r* = 0.05) consist of the same values of the odds ratio.

	r	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95
p	**odds**	0.05	0.11	0.25	0.43	0.67	1.00	1.50	2.33	4.00	9.00	19.00
0.05	0.05	**1.00**	0.47	0.21	0.12	0.08	0.05	0.04	0.02	0.01	0.01	0.00
0.10	0.11	2.11	**1.00**	0.44	0.26	0.17	0.11	0.07	0.05	0.03	0.01	0.01
0.20	0.25	4.75	2.25	**1.00**	0.58	0.38	0.25	0.17	0.11	0.06	0.03	0.01
0.30	0.43	8.14	3.86	1.71	**1.00**	0.64	0.43	0.29	0.18	0.11	0.05	0.02
0.40	0.67	12.67	6.00	2.67	1.56	**1.00**	0.67	0.44	0.29	0.17	0.07	0.04
0.50	1.00	19.00	9.00	4.00	2.33	1.50	**1.00**	0.67	0.43	0.25	0.11	0.05
0.60	1.50	28.50	13.50	6.00	3.50	2.25	1.50	**1.00**	0.64	0.38	0.17	0.08
0.70	2.33	44.33	21.00	9.33	5.44	3.50	2.33	1.56	**1.00**	0.58	0.26	0.12
0.80	4.00	76.00	36.00	16.00	9.33	6.00	4.00	2.67	1.71	**1.00**	0.44	0.21
0.90	9.00	171.00	81.00	36.00	21.00	13.50	9.00	6.00	3.86	2.25	**1.00**	0.47
0.95	19.00	361.00	171.00	76.00	44.33	28.50	19.00	12.67	8.14	4.75	2.11	**1.00**

'Minimum pH' (OR = 45.307). An increase of approximately 1.20 units in 'Minimum pH' is enough to compensate for the decrease caused by the change in fertility requirement (as $45.307^{1.204} = 55.56$).

Other ways to compare proportions

Odds ratio is not the only measure to compare proportions, as we have already noted. Two other commonly used measures are the ratio of proportions (known as the risk ratio, RR) and the difference of proportions (DP) (Cox & Snell 1989, Fleiss 1994). In the medical literature, there has been considerable debate about the interpretation, as well as the use and misuse, of the different effect measures (Walter 2000, Holcomb *et al*. 2001, Case *et al*. 2002, Sistrom & Garvan 2004). Most of this debate is concerned with the correct interpretation of different measures and not their statistical properties *per se*. All measures have their weaknesses and strengths and thus their applicability varies across situations. OR, however, has most of the statistical properties that an ideal effect measure should have and it is therefore more suitable for a variety of study designs and data analyses (Cox & Snell 1989, Walter 2000). Although OR may be intuitively less suitable for public communication of research results, it is superior in the analysis of data (Walter 2000).

In ecology, the debate about different effect measures, particularly the relationship between OR and RR, has not been topical. However, it is worth pointing out here that OR and RR are mathematically related:

$$RR = p/r = OR/[(1 - r) + r \times OR]. \quad (4)$$

From this it results that OR and RR are close to each other when the reference proportion r is small (< 0.05), but may become very different as r increases (Holcomb *et al*. 2001, Sistrom & Garvan 2004). For example, for $p = 0.1$ and $r = 0.05$, RR = 2, whereas OR = 2.111, but for $p = 0.5$ and $r = 0.25$, RR = 2, whereas their odds ratio equals 3. If $p = 1.0$ and $r = 0.5$, RR equals 2, but OR is infinite!

Table 3. Reporting OR as a value above 1 together with the exponent 1 or –1 to denote the direction of the effect helps the comparison of the effect sizes between suppressing and promoting factors. The OR's in the 'Original' column characterize the effect of various factors associated with the ability of exotic woody plants in New Hampshire to naturalize [modified and extended from Table 3 in Frappier & Eckert (2003)]. Without the inverse values (given in the 'Preferred' column), it is not obvious that, for example, the effect of 'Individual growth rate' (OR = 8.274), roughly equals the effect of 'Shade tolerance' (OR = 0.129), but in the opposite direction.

	OR		Logit difference	Standard error	Confidence interval	
	Original	Preferred			Logit	OR
Conifer	0.001	1000^{-1}	–6.908	2.816	–12.43–(–1.39)	249475^{-1}–4.01^{-1}
Fertility requirement	0.018	55.56^{-1}	–4.017	1.531	–7.02–(–1.02)	1117^{-1}–2.76^{-1}
Minimum pH	45.307	45.307	3.813	1.779	0.33–7.30	1.39–1481
Ability to resprout	21.174	21.174	3.053	1.803	–0.48–6.59	1.62^{-1}–725
Individual growth rate	8.274	8.274	2.113	0.973	0.21–4.02	1.23–55.71
Shade tolerance	0.129	7.751^{-1}	–2.048	1.068	–4.14–0.05	62.88^{-1}–1.05
Fire tolerance	0.187	5.347^{-1}	–1.677	0.828	–3.30–(–0.05)	27.10^{-1}–1.06^{-1}
Minimum temperature	0.699	1.430^{-1}	–0.358	0.133	–0.62–(–0.10)	1.86^{-1}–1.10^{-1}
Native latitudal range	1.398	1.398	0.335	0.142	0.06–0.61	1.06–1.85
Fruit size	1.090	1.090	0.086	0.043	0.00–0.17	1.00–1.19
Minimum density	1.008	1.008	0.008	0.003	0.00–0.01	1.00–1.01

Logit difference is the natural logarithm (ln) of OR. The 95% confidence interval for logit difference is calculated as $\ln(OR) \pm 1.96 \times SE$, which for e.g. Ability to resprout gives $3.053 \pm 1.96 \times 1.803 = [-0.481, 6.587]$. By taking antilog of the resulting lower and upper limits one obtains the confidence interval for OR: [0.618, 726]. Note that the confidence interval is not symmetrical with respect to the observed OR (21.174) because of the asymmetrical nature of the odds ratio scale.

Conclusions

Our exposé has hopefully revealed some of the practical and theoretical advantages of the odds ratio as a measure of discrepancy between proportions. OR is the pivotal concept in the simple analyses of proportions in two-way contingency tables, as well as in complex logit model approaches. We feel that while there is a rather solid understanding among ecologists regarding how to deal with numerical response variables, the analysis of proportions — and their theoretical counterparts, probabilities — have proven trickier. As we have shown here, OR is a good way to handle proportions in that it — in addition to solving technical problems in statistics — supports powerful ecological interpretations. Because logit models are already widely used in ecological research, the associated statistical measure — odds ratio — should become a standard part of reporting in research papers.

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